Conference "Around forms, cycles and motives"
On the occasion of Albrecht Pfister's 80th birthday
Mainz, September 8–12, 2014

Luca Barbieri-Viale (Milan): *The motivic topos*

After recalling Nori's category of effective homological mixed motives I define its thick abelian subcategory of Nori n-motives. I then show how Nori 1-motives can be regarded as a full subcategory of an étale version of Voevodsky motives. I further discuss a topos-theoretic approach to the problem of classifying (co)homology theories versus Nori's formalism.

Jean-Benoît Bost (Orsay): *Constructing algebraic surfaces and vector bundles over number fields from formal and analytic data*

In this talk, I will discuss some constructions of algebraic surfaces over number fields and of vector bundles over such surfaces, starting from formal and analytic data. These constructions involve some kind of "infinite dimensional hermitian vector bundles over arithmetic curves" and related results of "infinite dimensional geometry of numbers".

Jean-Louis Colliot-Thélène (Orsay): *Rund um die unverzweigte Kohomologie herum*

Die Brauergruppe von Varietäten kann man zu verschieden Zwecken benutzen:
– Zeigen, daß eine komplexe Varietät nicht rational ist, d. h., ihr Funktionenkörper nicht rein transzendant ist (Artin-Mumford Invariante).
– Wenn der Grundkörper endlich ist, die Picardgruppe, d.h. die Klassengruppe von Divisoren, untersuchen (Tatesche Vermutung).
– Wenn der Grundkörper ein Zahlkörper ist, das Hasseprinzip und die schwache Approximation bei linearen algebraischen Gruppen studieren (Brauer-Maninsche Hindernis).

Unverzweigte Kohomologiegruppen sind Verallgemeinerungen der Brauergruppe. Ich werde beschreiben, wie man die dritte unverzweigte Kohomologiegruppe von Varietäten bei konkreten Problemen auch benutzen kann: Rationalitätsprobleme, Ganzzahlige Hodgesche Vermutung, Ganzzahlige Tatesche Vermutung, und Arithmetik von linearen algebraischen Gruppen über dem Funktionenkörper einer Kurve über einem p-adischen Körper.

Charles De Clercq (Paris 13): *Motivic equivalence of semisimple algebraic groups*

Two semisimple algebraic groups are said to be motivic equivalent if the motives of the associated projective homogeneous varieties are isomorphic. We will show in the lecture how the motivic equivalence of two such groups can be characterized by some coloured Dynkin diagram, in the same fashion as Tits' classification of algebraic groups. We will thereafter discuss consequences of this classification on birational geometry of some projective homogeneous varieties, as well as its link with other classical invariants of semisimple algebraic groups.

Olivier Haution (Munich): *Involutions of varieties and Rost's degree formula*

To an algebraic variety equipped with an involution, I will associate a cycle class in the Chow group of its fixed locus. I will give various properties of this class, and deduce a proof of the degree formula in arbitrary characteristic. Time permitting, I will use the notion of movable cycle classes to discuss the birational invariance of the top Segre class (modulo two).

Detlev Hoffmann (Dortmund): *Pfister and his forms*

In this talk, I will give a survey of some aspects of Pfister's work, in particular the theory of what are now called Pfister forms, and highlight the lasting influence of this work in the development of the algebraic theory of quadratic forms and related areas.

Annette Huber (Freiburg): *Motives of commutative group schemes and the motivic polylog*

(jt. work with G. Ancona, S. Enright-Ward, G. Kings, S. Pepin Lehalleur) Let S be a noetherian scheme of finite dimension, G/S a smooth commutative group scheme. We decompose the motive of G in the triangulated category of motives on S with rational coefficients into Kuenneth-components:

$$M_S(G) = \bigoplus M_i(G)$$

Of particular interest is the object $M_1(G)$ which is explicitly given by the sheaf G itself.

We apply this to give a definition of the motivic polylog for G. It is a class

$$\mathrm{pol} \in \mathrm{Ext}_S^{2d-1}(\mathcal{H}, \pi_! j_* \mathcal{L}og(d))$$

with $\mathcal{H} = M_1(G)[-1]$ and $\mathcal{L}og$ on G the motivic logarithm sheaf. It is an iterated extension of symmetric powers of $\mathcal{H}$. This generalizes the classical cases where G is the multiplicative group or an abelian scheme. Even in these cases, the construction is made simpler and more transparent by the use of relative motives.

Nikita Karpenko (Edmonton): *Incompressibility of products of Weil transfers of generalized Severi-Brauer varieties*

We generalize the incompressibility result of [1], concerning Galois Weil transfer of generalized Severi-Brauer varieties, to direct products of varieties of such type; as shown in [1], this is needed to understand essential dimension of representations of finite groups. We also provide a generalization to non-Galois (separable) Weil transfer.

[1] Karpenko, N. A., and Reichstein, Z. A numerical invariant for linear representations of finite groups. Linear Algebraic Groups and Related Structures (preprint server) 534 (2014, May 15, revised: 2014, June 18), 24 pages.

Moritz Kerz (Regensburg): *Specialization of zero cycles*

We study zero cycles on varieties over a local field. Saito and Sato obtained results about these zero cycles in the case of finite residue fields by looking at one cycles on a regular model and their étale cycle map and specialization. In the talk we will see how to generalize some of their results to arbitrary residue fields.

Roland Lötscher (Munich): *On gerbes banded by groups of multiplicative type*

For any algebraic group G over a field F there is an associated algebraic stack over F. It is denoted BG and called the classifying stack of G. The groupoid $BG(K)$ of objects over a field extension K/F is the category of G-torsors over K. Twisted forms of classifying stacks of algebraic groups are called gerbes. A typical example of a gerbe arises from a surjective homomorphism of algebraic groups $G \to H$. Namely if E is any H-torsor over F, there is a gerbe X_E such that the groupoid $X_E(K)$ is the category of liftings of E to a G-torsor. This gerbe becomes isomorphic to BN over any splitting field of E, where N is the kernel of $G \to H$. However in contrast to BN it may not have any object over F. In this talk we study gerbes over F banded by groups of multiplicative type. In particular we will study their essential dimension and canonical dimension and the relation between these two invariants.

Alexander Merkurjev (UCLA): *Motivic decomposition of compactifications of certain group varieties*

Let D be a central simple algebra of prime degree over a field and let G be the algebraic group of norm 1 elements of D. We determine the motivic decompositions of G and certain smooth compactifications of G. We also compute the Chow ring of G.

Ivan Panin (St. Petersburg): *A survey of recent results concerning a conjecture of Serre and Grothendieck*

The conjecture due to Serre and Grothendieck says that for a regular local ring R and a reductive R-group scheme G any principal G-bundle E over R which is trivial over the fraction field of R is trivial already over R.

The most recent result concerning this conjecture states that the conjecture holds for regular local rings containing a field. If the field is infinite then that is a result of a joint work due to Fedorov and the speaker (2012). If the field is finite then the result is due to the speaker (2014). A short survey of results on the topic will be given. Some ideas of the proof will be explained.

Victor Petrov (St. Petersburg): *A new cohomological invariant for groups of type* E_7 *arising from the Killing form*

We provide a new construction of Lie algebras of type E_7 over a field F; up to odd degree extension any Lie algebra of type E_7 with trivial Tits algebras arises this way. Computing the Killing form one gets an invariant in the Galois cohomology $H^5(F, \mathbb{Z}/2)$.

Alena Pirutka (EP Paris): *Stable invariants and quartic threefolds*

In a context of the Lüroth problem, for K a function field of a smooth projective variety X over a field k, one can ask whether K is purely transcendental over k (resp. a subfield of a purely transcendental extension of k, resp. becomes purely transcendental after adding some independent variables), that is, if X is rational (resp. unirational, resp. stably rational). In this talk we will discuss various invariants which allow to answer this questions for some classes of varieties, and more specifically, for quartic threefolds. By a celebrated result of Iskovskikh and Manin, no smooth quartic hypersurface in $\mathbb{P}^4_{\mathbb{C}}$ is rational. Using a specialization method introduced by C. Voisin, as well as a method based on the universal properties of the Chow group of zero-cycles, we will show that a lot of such quartics are not stably rational. This is a joint work with J.-L. Colliot-Thélène.

Markus Rost (Bielefeld): *On algebras of low degree*

The simplest example is the equivalence of commutative algebras of rank 3 and cubic binary forms. This correspondence exists over arbitrary rings and is described in terms of a tensor construction similar as for Clifford algebras. There are variations: Commutative algebras of rank 4, their cubic resolvents, quadratic algebras of low rank, the algebra of a binary polynomial and more.

Stephen Scully (MPI Bonn): *Splitting patterns of totally singular forms of prime degree*

In the past two decades, the emergence of new tools with which to study algebraic cycles on projective homogeneous varieties has led to major progress on several outstanding problems concerning the splitting behaviour of quadratic forms over fields of characteristic different from 2. After discussing some of the highlights of this progress, I will describe recent work which seeks to develop a similar theory of splitting for Fermat-type forms of degree p in characteristic $p > 0$. Lacking sufficiently flexible algebro-geometric tools with which to study the (totally singular) zero loci of these forms, we are forced to adopt here a more elementary and ad hoc approach. Nevertheless, a strikingly similar picture emerges. New results include an analogue of Karpenko's theorem on the possible values of the first Witt index.

Ismaël Soudères (Osnabrück): *(co)Lie coalgebra and Bloch's cycle complex over the projective line minus three points*

In this talk, we will present the relation between the tannakian coLie coalgebra of mixed Tate motive over the projective line minus three points X and Bloch's cycle complex computing the higher Chow groups of X. Then we will show how two explicit families of algebraic cycles in this complex induce a families of mixed Tate motives generating the geometric part of the "motivic" coLie coalgebra.

Alexander Vishik (Nottingham): *Subtle Stiefel-Whitney classes*

This is a joint work with Alexander Smirnov. I will discuss the new "subtle" version of Stiefel-Whitney classes, which, in contrast to the one introduced by Delzant and Milnor, sees the powers of the fundamental ideal, as well as the Arason invariant and its higher analogues. The key here is the computation for the case of a Pfister form. Our classes are also related to the J-invariant of quadrics, and are essential in the motivic description of some homogeneous varieties associated to a quadratic form. They should serve as a building block in the new homotopic approach to the classification of torsors of an orthogonal group.

Kirill Zainoulline (Ottawa): *Cohomological Invariants and torsion in the Chow group of a twisted flag variety*

In the present talk we discuss connections between cohomological invariants of linear algebraic groups and algebraic cycles on twisted flag varieties. This talk is based on several recent results by the speaker, S. Garibaldi, A. Neshitov and A. Merkurjev.