Alexander Vishik

Growth rate of motivic cohomology and Elementary Discrete Invariant of quadrics

Abstract: Considered as an object of triangulated category of Voevodsky, the motive of a quadric can be presented as an extension of simple motives $\mathcal{X}_Q, \mathcal{X}_{Q^1}, \ldots, \mathcal{X}_{Q^d}$, where Q^i is the Grassmannian of i-dimensional projective planes on Q, and $\mathcal{X}_S$ is the motive of the Chech simplicial scheme associated with the pair $S \to \mathrm{Spec}(k)$. And the above set of objects knows everything about $M(Q)$ (and vice-versa). The motive $\mathcal{X}_Q$ is a “form” of Tate-motive: over $\overline{k}$ it becomes isomorphic to the motive of a point. But over the ground field it is (usually) infinite-dimensional. The question arises: how ”infinite” are these objects? This can be estimated by the ”rate of growth” of their motivic cohomology groups. It appears that the rate of growth of motivic cohomology of $\mathcal{X}_Q, \mathcal{X}_{Q^1}, \ldots, \mathcal{X}_{Q^d}$ carries the same information as the Elementary Discrete Invariant $EDI(Q)$ of our quadric (an invariant defined in terms of Chow groups of quadratic Grassmannians).

This provides a new point of view on $EDI(Q)$. An interaction between these two invariants gives interesting results about both. It also suggests an approach to the Elementary Discrete Invariant for the algebraic groups of other types.